

1. A transformation T from the z -plane to the w -plane is given by

$$w = \frac{z + 2i}{iz} \quad z \neq 0$$

The transformation maps points on the real axis in the z -plane onto a line in the w -plane.

Find an equation of this line.

(4)



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Question 1 continued

Q1

(Total 4 marks)



P 4 2 9 5 5 A 0 3 3 2

2. Use algebra to find the set of values of x for which

$$\frac{6x}{3-x} > \frac{1}{x+1}$$

(7)



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Question 2 continued



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Question 2 continued



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Question 2 continued

Q2

(Total 7 marks)



P 4 2 9 5 5 A 0 7 3 2

3. (a) Express $\frac{2}{(r+1)(r+3)}$ in partial fractions.

(2)

- (b) Hence show that

$$\sum_{r=1}^n \frac{2}{(r+1)(r+3)} = \frac{n(5n+13)}{6(n+2)(n+3)}$$

(4)

- (c) Evaluate $\sum_{r=1}^{100} \frac{2}{(r+1)(r+3)}$, giving your answer to 3 significant figures.

(2)



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Question 3 continued



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Question 3 continued



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Question 3 continued

Q3

(Total 8 marks)



P 4 2 9 5 5 A 0 1 1 3 2

4. Given that

$$y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 + 5y = 0$$

- (a) find $\frac{d^3y}{dx^3}$ in terms of $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ and y .

(4)

Given that $y = 2$ and $\frac{dy}{dx} = 2$ at $x = 0$

- (b) find a series solution for y in ascending powers of x , up to and including the term in x^3 . (5)



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Question 4 continued



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Question 4 continued



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Question 4 continued

Q4

(Total 9 marks)



P 4 2 9 5 5 A 0 1 5 3 2

5. (a) Find, in the form $y = f(x)$, the general solution of the equation

$$\frac{dy}{dx} + 2y \tan x = \sin 2x, \quad 0 < x < \frac{\pi}{2} \quad (6)$$

Given that $y = 2$ at $x = \frac{\pi}{3}$

- (b) find the value of y at $x = \frac{\pi}{6}$, giving your answer in the form $a + k \ln b$, where a and b are integers and k is rational.

(4)



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Question 5 continued



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Question 5 continued



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Question 5 continued

Q5

(Total 10 marks)



P 4 2 9 5 5 A 0 1 9 3 2

6. The complex number $z = e^{i\theta}$, where θ is real.

(a) Use de Moivre's theorem to show that

$$z^n + \frac{1}{z^n} = 2 \cos n\theta$$

where n is a positive integer.

(2)

(b) Show that

$$\cos^5 \theta = \frac{1}{16} (\cos 5\theta + 5 \cos 3\theta + 10 \cos \theta)$$

(5)

(c) Hence find all the solutions of

$$\cos 5\theta + 5 \cos 3\theta + 12 \cos \theta = 0$$

in the interval $0 \leq \theta < 2\pi$

(4)



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Question 6 continued



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Question 6 continued



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Question 6 continued

Q6

(Total 11 marks)



7. (a) Find the value of λ for which $\lambda t^2 e^{3t}$ is a particular integral of the differential equation

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = 6e^{3t}, \quad t \geq 0 \quad (5)$$

- (b) Hence find the general solution of this differential equation.

(3)

Given that when $t = 0$, $y = 5$ and $\frac{dy}{dt} = 4$

- (c) find the particular solution of this differential equation, giving your solution in the form $y = f(t)$.

(5)



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Question 7 continued



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Question 7 continued



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Question 7 continued

Q7

(Total 13 marks)



8.

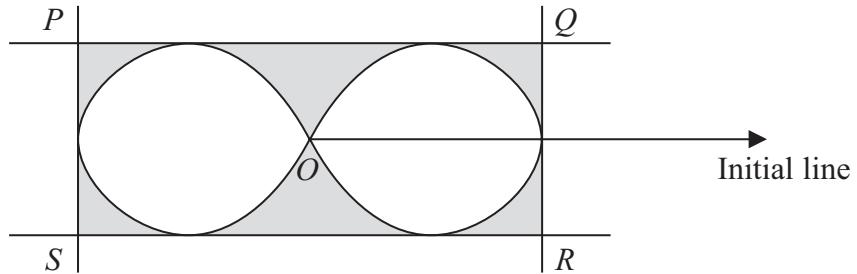


Figure 1

Figure 1 shows a closed curve C with equation

$$r = 3(\cos 2\theta)^{\frac{1}{2}}, \quad \text{where } -\frac{\pi}{4} < \theta \leq \frac{\pi}{4}, \quad \frac{3\pi}{4} < \theta \leq \frac{5\pi}{4}$$

The lines PQ , SR , PS and QR are tangents to C , where PQ and SR are parallel to the initial line and PS and QR are perpendicular to the initial line. The point O is the pole.

- (a) Find the total area enclosed by the curve C , shown unshaded inside the rectangle in Figure 1. (4)

(b) Find the total area of the region bounded by the curve C and the four tangents, shown shaded in Figure 1. (9)



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Question 8 continued



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Question 8 continued

Q8

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

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